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METHODS FOR SOLVING THE INTERSECTION OF ROTATION AREAS WITH STRAIGHT LINES

Abstract: *Given the frequent practical use of rotation areas, the paper proposes to study the methods for solving and determining the intersection of these rotation areas with various pipes, assimilated to the straight line.*

The areas considered for the study are the sphere, hyperboloid, ellipsoid and paraboloid. Thus, for the sphere are solved cases in which the straight line passes or not through the center of it, and for the hyperboloid cases when the straight line is or is not concurrent with its axis.

When determining the points of intersection between the rotation areas and straight lines are used descriptive geometry methods, the method of auxiliary cone and Rouche's method.

Key words: *sphere, hyperboloid, ellipsoid and paraboloid.*

1. INTRODUCTION

Rotation areas are commonly used in the technique. There are a lot of cases where they are crossed by various pipe, in different installations, whose parts are.

The paper proposes the assimilation of these pipes with straight lines and to study the possibilities to determine points of intersection between these pipes and different rotation areas.

2. THE INTERSECTION SPHERE – STRAIGHT LINE

Generally, a straight line intersects a sphere in two points. There are two cases: the straight line passes or not through the center of the sphere.

2.1 The intersection of the sphere with a straight line passing through the center of the sphere

In Figure 1, are determined points in which the straight line D(d,dô), which passes through the center of the sphere ɋ(ɤ,ɤô), intersects the sphere, using the rotation method.

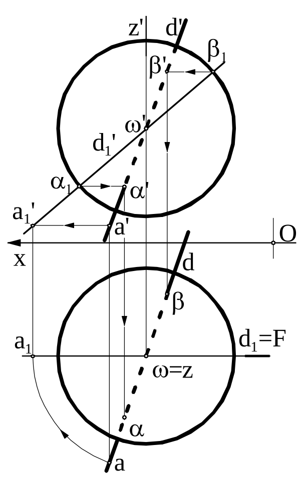

Fig. 1 The intersection of the sphere with a straight line passing through the center of the sphere

2.2 The intersection of the sphere with a straight line which does not pass through the center of the sphere

Determining points where a straight line which does not pass through the center of the sphere intersects the sphere can be done in four ways, as exemplified below.

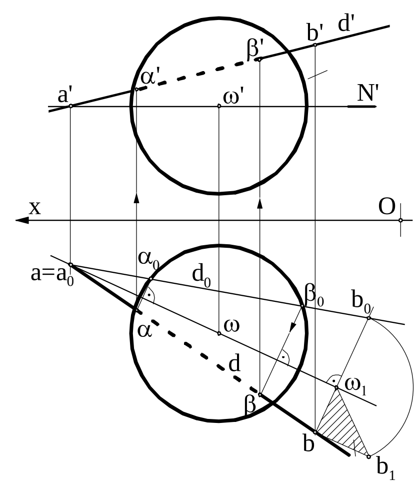

Fig. 2 The intersection of the sphere with a straight line which does not pass through the center of the sphere - using the rabattation on a level plane.

In Figure 2 is applied rabattation method on parallel planes with the projection planes. Thus it is considered the plane defined by the straight line D(d,dô) and the center of the sphere ɋ(ɤ,ɤô). Solving can also be made using rabattation method on a front plane drawn through the center of the sphere.

The rabattation method is also used in figure 3. It is considered the vertical plane [P], passing through the straight line D(d,dô), P ≡ d and it is rabattated the plane together with the straight line and the circular section, which is determined in the sphere, on horizontal plane of projection.

In Figure 4 is applied the change of vertical projection plane method.

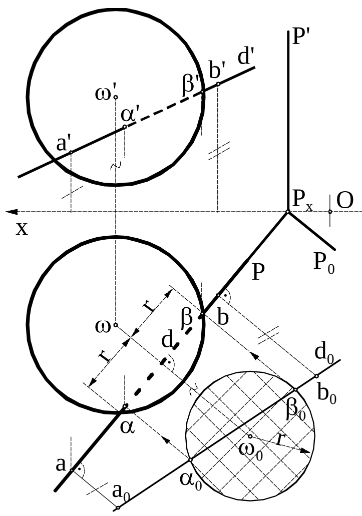

Fig. 3 The intersection of the sphere with a straight line which does not pass through the center of the sphere ï using the rabattation of vertical plane of the straight line.

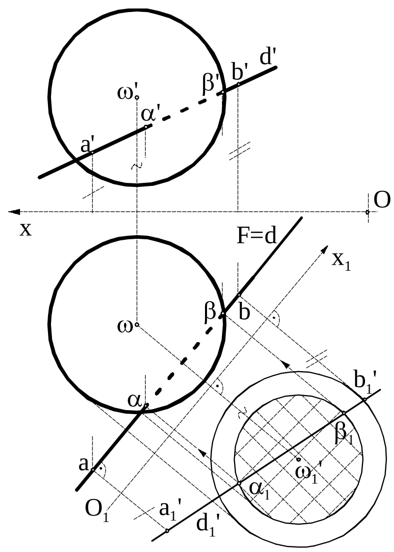

Fig. 4 The intersection of the sphere with a straight line which does not pass through the center of the sphere - using the change of vertical plane method.

To use auxiliary cone method it is considered a cone having as directrix curve, the equator circle of the sphere and the vertex, the point S(s,sô), located in front plane passing through the center of the sphere (fig.5). It determines the main meridian circle in vertical projection. The vertical projection sô of vertex of the cone is given by the intersection of cone apparent contour generators, of the vertical projection: eôaô ∩ fôbô = sô. Horizontal projection s of the vertex of cone is located on the horizontal trace of front plane F.

Since the edge view plane [Q] passed through the straight line d, dô=Qô, intersects the sphere and also the cone defined above by the same circle, the points of intersection between the straight line and the cone will be the same as with the sphere. Therefore are determined the points of intersection between the straight line D(d,dô) and the cone, using the method of longitudinal section. The plane passing through the vertex and the straight line, intersects the level plane [N] after the straight line Δ(δ,δô). It is determined by the points where the straight line D and the generators SA and SB intersects the level plane of the equator of the sphere: δ = h ∪ i. The longitudinal section determined in the cone is 1s2. It is intersected by the straight line d in points α and β, which are also points of intersection with the sphere.

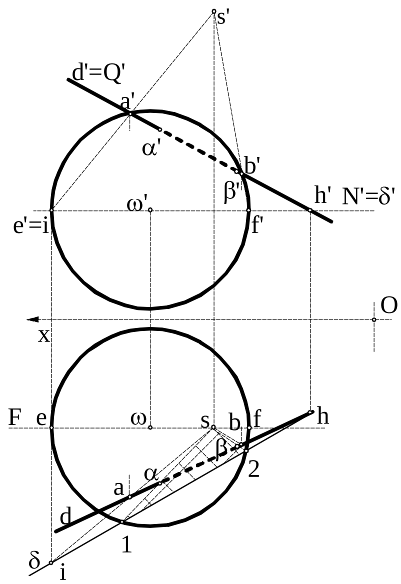

Fig. 5 The intersection of the sphere with a straight line which does not pass through the center of the sphere - using the auxiliary cone method.

3. THE INTERSECTION HYPERBOLOID – STRAIGHT LINE

3.1 Hyperboloid intersection with a straight line that is not concurrent with its axis

It is considered a hyperboloid given by its axis Z and the main generator CH (fig.6). The girdle circle is the circle of radius ɤ c and the second main generator C_1H_1.

The points of intersection between the straight line D(d,dô) and the hyperboloid, is determined by applying the transversal section method. Horizontal projection of the straight line intersects the section determined in the hyperboloid by an edge view plane passed through the straight line. The section is an ellipse of axes 12 and 56, having points of intersection with the girdle circle, points 3 and 4. Since the elliptical section is passed through points, the points of intersection α and β between the straight line and hyperboloid, are determined with some errors.

A more precise determination is obtained using the Rouche method. This is based on the theorem according to which two quadrics of revolution with parallel axes and with the same plane of symmetry perpendicular to the axis, intersects following a curved line, which projection on the common plane of symmetry or on a parallel plane with it is a circle.

This may be demonstrated considering the equations of the two quadrics as being:

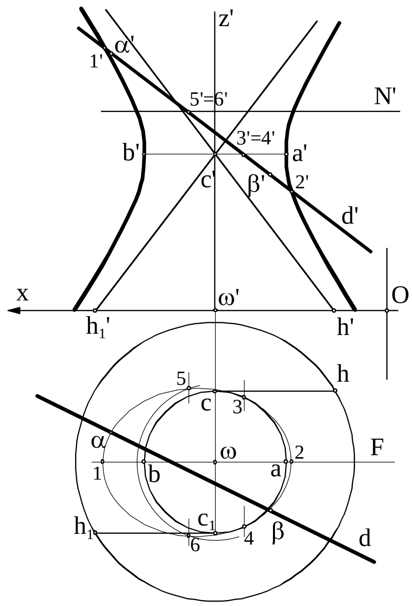

Fig. 6 Hyperboloid intersection with a straight line that is not concurrent with its axis ï transversal section method.

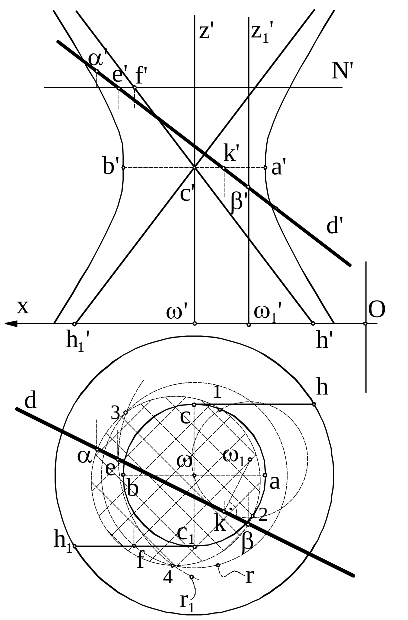

Fig. 7 Hyperboloid intersection with a straight line that is not concurrent with its axis ï Rouche method

$$A(x^2+y^2) + z^2 + 2Bx + 2Cy + D = 0$$
$$D(x^2+y^2) + z^2 + 2Ex + 2Fy + G = 0$$

Subtracting between them is obtained the equation of the projection of intersection curved line on a perpendicular plane on their axis:

$$(A - D)(x^2+y^2) + 2(B - E)x + 2(C - F)y + (D - G) = 0$$

which is the equation of a circle.

In Figure 7, it is considered a second hyperboloid, along with the given hyperboloid, which has the same girdle plan as the first one. It is generated by the straight line D, rotating around the vertical axis Z_1, which intersects the horizontal plane at point ɋ$_1$(ɤ$_1$,ɤ ô$_1$). This point is on the perpendicular line raised from point k, which is the horizontal projection of point of intersection between straight line D and the girdle plane of the given hyperboloid.

According to the theorem previously mentioned, the projection of the curved line of intersection between the two hyperboloids on the horizontal plane is a circle. Two points of it are points 1 and 2 of intersection between the two girdle circles of the two hyperboloids. To determine other points of the circle are drawn another two circles resulting by sectioning the hyperboloids with a level plane [N]. Hyperboloid of Z axis is intersected by the level plane [N] after the circle r, of radius ɤe and hyperboloid of Z_1 axis, after the circle r_1, of radius ɤf. The two circles intersect in points 3 and 4. The points 1, 2, 3 and 4 determine the searched circle, which is intersected by the straight line D in points (α,αô) and (β,βô), the points of intersection between the straight line and hyperboloid.

3.2 Hyperboloid intersection with a straight line which is concurrent with its axis

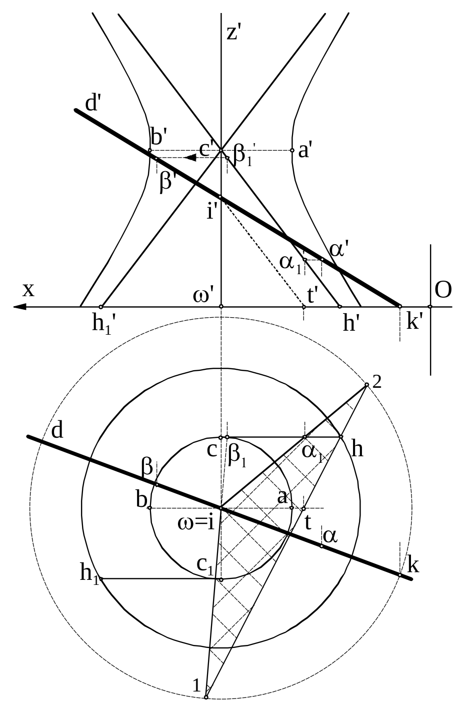

Fig. 8 Hyperboloid intersection with a straight line which is concurrent with its axis

The points of intersection between the concurrent straight line and the hyperboloid axis from Figure 8 is determined considering the cone generated by the straight line D, rotating around hyperboloid axis. This cone has the vertex at point I(i,iô) of intersection between the straight line D and hyperboloid axis. The cone intersects the hyperboloid by two circles, which are parallel to the horizontal plane, so they are in level planes. These circles intersection with the straight line will determine the points of intersection with the hyperboloid. The base of cone is the circle of radius ɤk. Through the vertex of cone from vertical plane, iô, it is drawn a parallel line to the hyperboloid generator, iα̂ô || cα̂hô, respectively, it || ch and it is determined point tô, respectively, t. Points t and h determine the horizontal trace of the plane which intersects the cone (P = t ∪ h) longitudinally, by the generators i1 and i2. Hyperboloid generator ch intersects the triangular section 1i2 of the plane in points α_1 and β_1, which have vertical projections points α_1ô and β_1α̂. Through points α_1ô and β_1ô are passed circles of intersection between the cone and hyperboloid (parallel to the horizontal plane), to these quotas being determined, on the vertical projection of straight line dô, points αô and βô and also points α and β, which are the points of intersection between the straight line and hyperboloid.

4. THE INTERSECTION PARABOLOID – STRAIGHT LINE

It is considered paraboloid of rotation in Figure 9, given by the vertical axis Z(z,zô) and the principal meridian parable in front plane [F]. To determine the points of intersection between the straight line D(d,dô) and the paraboloid it is applied the method of transversal sections, passing the edge view plane [Q], through the straight line. It intersects the paraboloid as an ellipse, which is projected on the vertical plane by the segment aα̂bô, and on the horizontal plane, by the circle of ab diameter. The intersection of the horizontal projection d with this circle represents points of intersection with the paraboloid, α and β, respectively αô, βô in vertical plane.

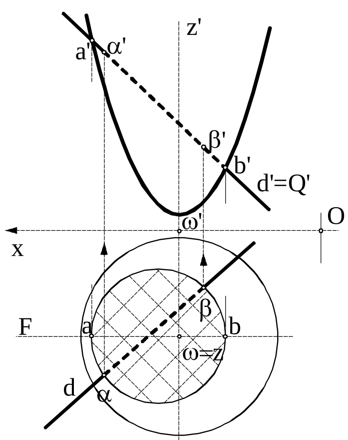

Fig. 9 Paraboloid intersection with a straight line

5. THE INTERSECTION ELLIPSOID – STRAIGHT LINE

The points of intersection between the ellipsoid of rotation in Figure 10 and straight line D(d,dô) is determined by applying the method of auxiliary cone. This cone is defined by the directrix curve, which is the ellipse mα̂nô and the vertex S(s,sô), located on the major axis of the ellipsoid. The ellipse whose vertical projection is mα̂nô segment, is the section made of an edge view plane [Q], in the ellipsoid, drawn through the straight line, dô = Qα̂. Auxiliary cone mα̂sα̂nô intersects the horizontal plane by the circle of AB diameter. Points of intersection between the straight line D(d,dô) and the ellipsoid will be the same with points where the straight line intersects the cone defined above. It is considered another straight line Δ(δ,δô), drawn through the cone vertex, parallel to the straight line D. The plane defined by the straight lines D and Δ, whose horizontal trace is P, P = h ∪ h_1, sectioned longitudinally the cone by the triangle 1s2. The cone generators s1 and s2 intersects the straight line d in points α and β, which also represents the points of intersection with the ellipsoid.

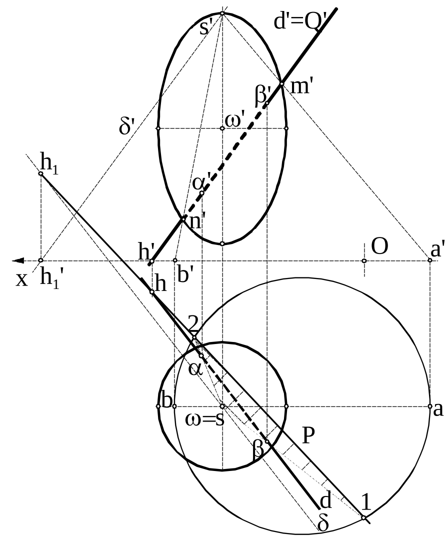

Fig. 10 Ellipsoid intersection with a straight line

6. CONCLUSIONS

This paper summarizes the possibilities of determining the points of intersection between the straight line and rotation areas. Were used the three methods of descriptive geometry, the method of changing projection planes, rotation method and rabattation method, as well as other related methods, such as the method of auxiliary cone and Rouche method.

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