Isidora MITROVIĆ, Sonja KRASIĆ, Jovana TOMIĆ, Nastasija KOCIĆ

CONSTRUCTION OF THREE-DIMENSIONAL MODEL OF PLATONIC SOLIDS UTILIZING METATRON'S CUBE

Abstract: *The concept of Metatron's Cube, which is rooted in Sacred Geometry for centuries, is believed to contain the projections of all Platonic Solids. While Metatron's Cube inherently manifests as a two-dimensional geometric form, this study incorporates its three-dimensional counterpart delving into the relationship between this sacred symbol and the Platonic Solids. The primary objective is to ascertain the viability of constructing the five Platonic Solids exclusively through their Metatron's Cube projections by experimenting in the fields of descriptive geometry and geometry in general.*

The experimentation strategically emphasizes simplicity and feasibility, seeking to unveil potential methodologies for constructing these Platonic Solids via the projections engendered by Metatron's Cube. Through the establishment of a connection between theoretical frameworks and practical applications, the study aims to contribute valuable insights into alternative and probably easier and more accessible methods of constructing Platonic Solids, particularly within the context of architectural design. This research represents a valuable stride in enhancing our understanding of ancient geometric concepts and their potential applications in contemporary architectural practices.

Key words: *Metatron's Cube, Platonic Solids, descriptive geometry, three-dimensional model construction*

1. INTRODUCTION

According to Sacred Geometry, Metatron's Cube is considered a perfect geometrical form due to its symmetrical arrangement and intricate patterns. [1] From a purely geometric perspective, Metatron's Cube showcases principles of symmetry, proportion, and interconnectedness, making it an intriguing object of study for mathematicians and geometry enthusiasts. [2] [3] While Metatron's Cube is a two-dimensional representation, it contains the potential for constructing the three-dimensional Platonic solids utilizing its structure. Despite Sacred Geometry is considered an ancient science that explores and explains the geometrical forms, shapes, patterns, and fractals, it is more focused on symbolism and spiritual significance across various cultures and traditions. Correspondingly much remains uncharted and necessitates verification concerning the theory of geometry as a science.

The main premise of this study posits that the three-dimensional Metatron's Cube contains all Platonic Solids. To substantiate or refute this hypothesis, an ancillary conjecture is proposed: that the two-dimensional Metatron's Cube encapsulates a projection of each Platonic Solid.

2. METHODS

The initial approach adopted in this study involved data collection, followed by their analysis. The collected information from literature and data was quite limited and insufficiently relevant, given that the Metatron's Cube is more commonly examined from the perspectives of art, symbolism, and its effects on humans rather than from a geometrical standpoint. Each piece of information had to be verified and tested through experimentation.

Based on the findings, two hypotheses were formulated at the beginning of the study, one primary and one secondary, some of which were confirmed while others were refuted in various instances.

The primary focus of the study revolved around experimentation and independent search for clarification of the set hypotheses. This exploration was based on creating certain models: both the two-dimensional and three-dimensional representations of the Metatron's Cube, as well as models of Platonic Solids and their mutual overlap. The construction of these models included the utilization of AutoCAD software. Three-dimensional construction of Metatron's Cube was challenging due to lack of literature connected to this topic. Through a synthesis of existing and newly acquired data, a cohesive framework was established to ultimately reach conclusions.

3. METATRON'S CUBE

Metatron's Cube as a two-dimensional geometric figure is composed of thirteen equal-sized circles which are symmetrically arranged. Each circle's centre is connected to every other circle's centre by straight lines, resulting in a complex network of interlocking shapes. [4] Three-dimensional Metatron's Cube is geometric figure which is composed of seventeen equal-sized spheres connected to each other's centres by straight lines.

3.1 Construction of Metatron's Cube in two dimensions

Two-dimensional construction of Metatron's Cube started with drawing an initial circle and afterwards drawing two belts of additional six circles each, later called the inner and the outer circles. The circles were arranged to connect with one another, with their centres positioned along straight lines intersecting at a 60° angle at the centre of the initial circle. The inner belt of six circles is arranged so that the initial circle touches at points on intersecting straight lines at a 60° angle. The outer belt of six circles touches the inner belt at points along the intersecting lines at a 60° angle (Figure 1. left).

The final step was to connect all the centres of the circles to each other by drawing straight lines (Figure 1. right). In this way two-dimensional construction of Metatron's Cube was created.

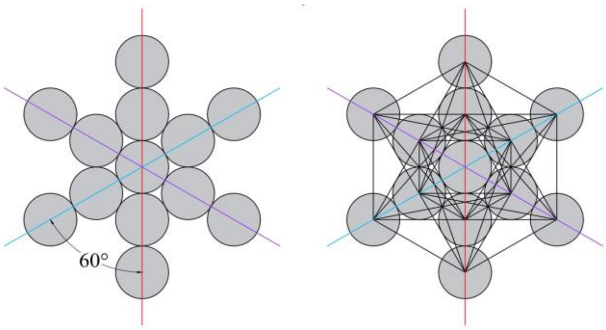

Figure 1 Construction of two-dimensional Metatron's Cube.

3.2 Construction of Metatron's Cube in three dimensions

To facilitate the construction of the Metatron Cube, it was essential to create an auxiliary geometric figure known in Sacred Geometry as Merkaba. Different name for this geometric figure is stellated octahedron. This figure represents two tetrahedron intersecting each other perfectly. It also can be explained like two interlocking regular triangular based pyramids, which touches each other with bases and both tops are on different sides.

For creating Merkaba, three circles' centres from outer belt of circles of two-dimensional model of Metatron's Cube were used as a base of one tetrahedron with one apex facing upwards (Figure 2. left). Other three outer circles centres were used to create a base of the other tetrahedron with one apex facing downwards (Figure 2. right).

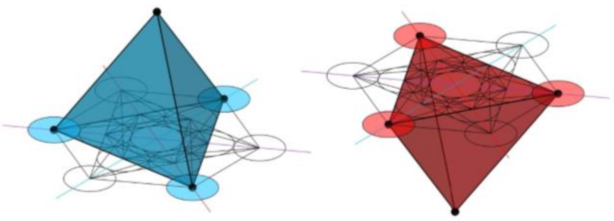

Figure 2 Construction of two tetrahedrons.

This two tetrahedrons were created using the same two-dimensional model of Metatron's Cube (Figure 3. left). Afterward, one of the tetrahedrons was moved vertically to create symmetrical overlap. This step could be done in two different ways which resulted constructing the same shape. Either the tetrahedron which one apex is facing upwards was moved down vertically, or the tetrahedron with one apex is facing downwards was moved up vertically. It is important that this step is done in the way that the bases of both tetrahedrons intersect the edges of the other one at their midpoint (Figure 3. right).

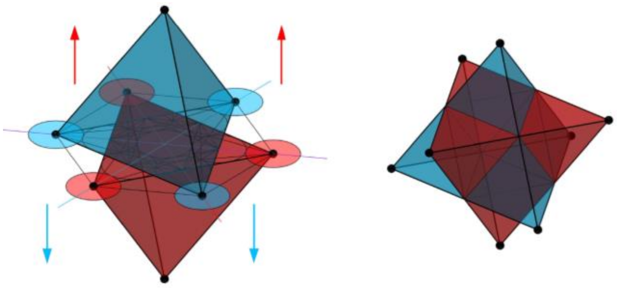

Figure 3 Construction of Merkaba using two tetrahedrons.

By connecting the vertices of the Merkaba by straight lines, axes were obtained, forming a structure resembling a coordinate system. Within the Merkaba's centre, where the axes intersect, the initial sphere with the same diameter as the circles in two-dimensional model of Metatron's Cube was placed. At each Merkaba vertex, a sphere of identical diameter to the central one was placed, resulting in the formation of the outer belt of eight spheres. (Figure 4. left). Additionally, eight spheres were placed midway between the centre of the initial sphere and each of the centres of outer spheres (corresponding to each Merkaba's vertex) all sharing the same diameter as the initial and outer-belt spheres. Therefore, the inner belt of eight spheres was formed. Ultimately, the centres of each sphere were interconnected, leading to the finalization of the three-dimensional Metatron's Cube (Figure 4. right).

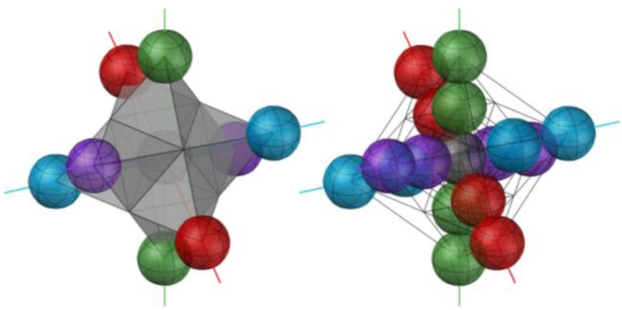

Figure 4 Construction of three-dimensional Metatron's Cube utilizing the shape of Merkaba.

Following the creation of the three-dimensional model, it was determined that the angular distances between each of the established axes were approximately 109.47° and 70.53°. Angles between these axes were measured using command "Measure" in AutoCAD software. Through simple observation, it was deduced that despite the circles touching within the two-dimensional Metatron's Cube, spheres of equivalent diameter within the three-dimensional Metatron's Cube remained without common point of intersection. Furthermore, by aligning the perspective perpendicular to each axis, it was determined that the two-dimensional projection of the Metatron's Cube is not merely the top and bottom view of the three-dimensional model, but indeed represents a projection with a total of eight distinct sides.

4. CONSTRUCTION OF EACH OF THE PLATONIC SOLIDS

In following chapters, it was examined whether it is possible to construct each of the Platonic Solids using the Metatron's Cube's three-dimensional model directly or indirectly. For each of the Platonic Solids, the initial inquiry centered around determining if its projection was contained within the two-dimensional model of the Metatron's Cube, establishing a criterion for its three-dimensional construction.

4.1 Tetrahedron

A tetrahedron is a regular polyhedron consisting by four faces, each of which is an equilateral triangle. This Platonic Solid is defined by four vertices and six edges. [5] [6] Considering that the tetrahedron was needed to create the three-dimensional Metatron's Cube, it was straightforward to conclude the feasibility of constructing a tetrahedron from the three-dimensional model of the Metatron's Cube.

Within the two-dimensional projection of the Metatron's Cube, the equilateral triangle was clearly observed, which simultaneously represented one of the projections of the tetrahedron. This triangle was created by connecting the centres of every other circle from the outer belt of circles using equal-dimension straight lines. (Figure 5). The angle between these lines representing the sides of the triangle is 30°, which proves that the triangle is equilateral.

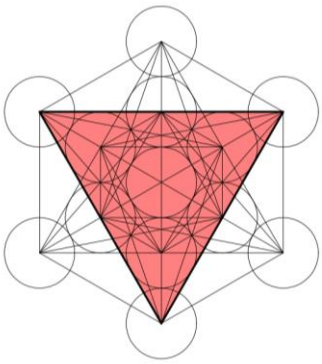

Figure 5 Triangle as a projection of tetrahedron in two-dimensional Metatron's Cube

Except the triangle, the other projection of tetrahedron was observed. This projection of a tetrahedron was formed by linking the centres of every other circle from the outer belt of circles using straight lines of equal dimensions, followed by the addition of three more straight lines connecting the centre of the initial circle with the three previously used centres of every other outer circle (Figure 6). In accordance with this, it was concluded that there was a possibility of creating a three-dimensional model of this Platonic Solid.

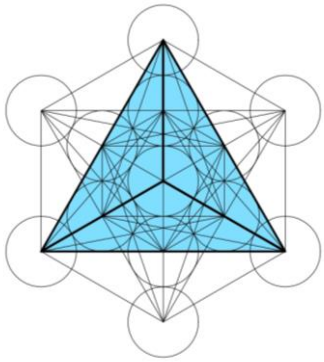

Figure 6 Projection of tetrahedron in two-dimensional Metatron's Cube

For constructing the three-dimensional model of the tetrahedron, three other outer spheres' centres were connected via straight lines, forming an equilateral triangle. Further, these centres were then connected to the centre of the furthest sphere, resulting in the formation of three more equilateral triangles. Through this process, the Platonic solid - the tetrahedron, was constructed (Figure 7). Within the three-dimensional Metatron's Cube, it was possible to construct a tetrahedron in two ways. The first method included creating the upper base by connecting the centres of three upper spheres from the outer belt of spheres, in the next step followed by connecting each of these centres to the centre of the lowest outer sphere (Figure 7. left). The second method is very similar and completely symmetrical. The lower base was created by connecting the centres of three lower spheres from the outer belt of spheres. In the final step, each of these centres were connected to centre of the uppermost outer sphere (Figure 7. right)

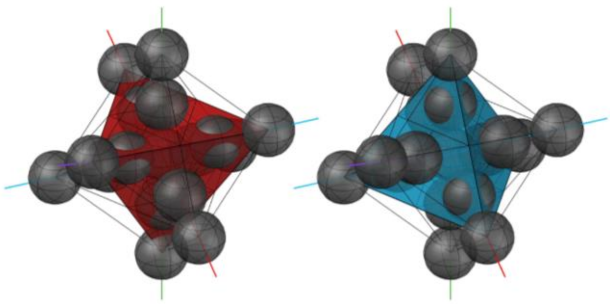

Figure 7 Tetrahedron construction utilizing three-dimensional Metatron's Cube

4.2 Hexahedron (Cube)

A hexahedron is a regular polyhedron consisting by six faces, each of which is a square. This Platonic Solid is defined by eight vertices and twelve edges. [5] [6]

One of the projections of the hexahedron (Figure 8) is contained within the two-dimensional Metatron's Cube. The initial step to construct this projection of hexahedron utilizing Metatron's Cube was to create a hexagon through the connection of adjacent circle centres, which belong to the outer belt of circles. This step is followed by the linkage of every second outer circle centre to the centre of the initial circle. Through this procedure, a projection of the hexahedron was formed. This observation led to the deduction that there is a potential for creating a three-dimensional model of this Platonic Solid.

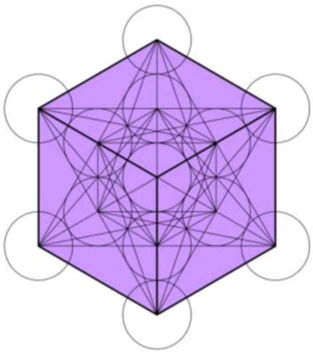

Figure 8 Projection of hexahedron in two-dimensional Metatron's Cube

Utilizing three-dimensional Metatron's Cube, hexahedron was created in the following way. Each centre of the spheres in the outer belt was connected with precisely three adjacent centres of the outer belt spheres (Figure 9). From this, it can be concluded that the centres of all circles in the outer belt represent the vertices of the cube, while the axes belonging to the Metatron's Cube correspond to the diagonals of the hexahedron.

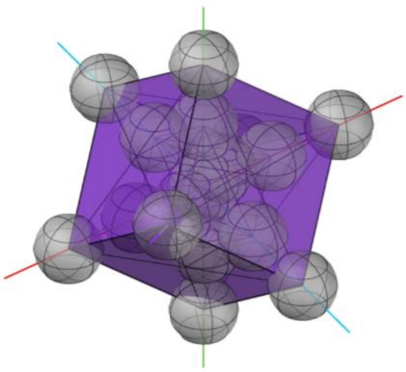

Figure 9 Hexahedron construction utilizing three-dimensional Metatron's Cube

4.3 Octahedron

An octahedron is a regular polyhedron consisting by six faces, each of which is an equilateral triangle. This Platonic Solid is defined by six vertices and twelve edges. [5] [6]

Within the two-dimensional Metatron's Cube, two identical projections of the octahedron are observed, with one being larger and the other one smaller. The larger projection was obtained by connecting the centres of each of the circles from outer belt forming hexagon. Afterwards, the centres of every other outer circle were connected to one another, forming an equilateral triangle (Figure 8. left). The smaller projection is obtained following the same process but utilizing the inner circles. Centres of each of the circles from inner belt were connected forming hexagon. Afterwards, the centres of every other inner circle were connected to one another, forming an equilateral triangle (Figure 8. right).

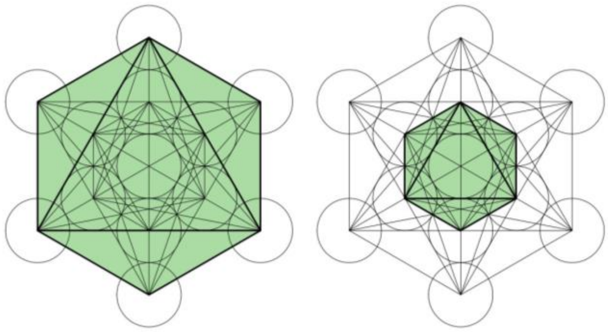

Figure 10 Projection of octahedron in two-dimensional Metatron's Cube, on the left: larger one using outer belt of circles, on the right: smaller one using inner belt of circles

Construction of the smaller octahedron, corresponding by size to the projection of the smaller octahedron within the Metatron's Cube, can be explained in several different ways, but it always involves the exact same points located within the framework of the three-dimensional Metatron's Cube (Figure 11).

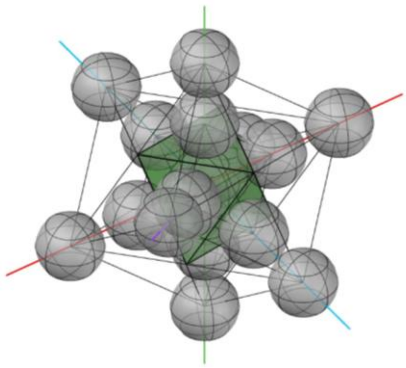

Figure 11 Smaller octahedron construction utilizing three-dimensional Metatron's Cube

The most understandable way is that an octahedron was obtained as the intersection of two tetrahedrons created within the three-dimensional Metatron's Cube. The midpoints of the edges of the tetrahedron represented the vertices of the octahedron. By connecting the midpoints of edges from these two tetrahedrons, the edges of the octahedron were formed.

Another way to explain the construction of the octahedron is through the hexahedron already constructed within the Metatron's Cube. Specifically, each point located at the intersection of the diagonals of the hexahedron's faces represents one of the vertices of octahedron. When this points were connected, the octahedron was formed.

Constructing the larger octahedron proved unfeasible within the confines of the existing spheres of the three-dimensional Metatron's Cube, thus its construction required the addition of two more outer belts of spheres. By adding this, the extended version of Metatron's Cube is formed. This construction of the larger octahedron can be explained in several different ways, but it always involves the exact same points located within the framework of the expanded three-dimensional Metatron's Cube (Figure 12).

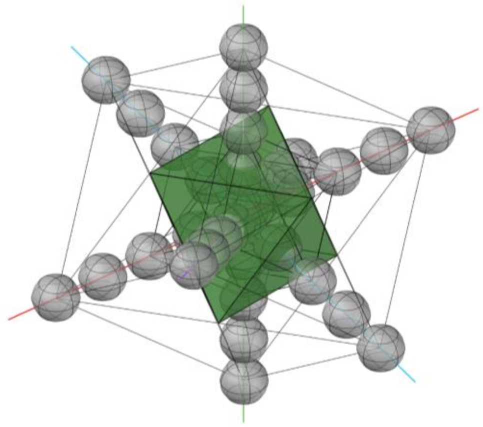

Figure 12 Larger octahedron construction utilizing expanded three-dimensional Metatron's Cube

Within this expanded Metatron's Cube, two larger tetrahedrons were formed using the last outer circles centres. Analogously to the construction of the smaller octahedron using two tetrahedrons from Metatron's Cube, the larger octahedron was constructed using two tetrahedrons from expanded Metatron's Cube.

Within the expanded Metatron's Cube, hexahedron was formed using the last outer circles centres. Analogously to the construction of the smaller octahedron using hexahedron from Metatron's Cube, the larger octahedron was constructed using hexahedron from expanded Metatron's Cube.

4.4 Dodecahedron

A dodecahedron is a regular polyhedron consisting by twelve faces, each of which is a pentagon. This Platonic Solid is defined by twenty vertices and thirty edges. [6]

Initially, the projection of the dodecahedron appeared to be present within the two-dimensional Metatron's Cube (Figure 13). Nevertheless, upon closer examination, it was determined that constructing this projection using points, lines, and circles as elements of the two-dimensional Metatron's Cube is not possible.

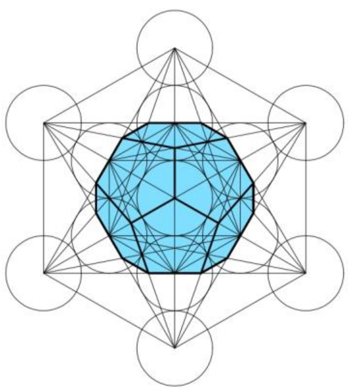

Figure 13 Dodecahedron look-alike shape in two-dimensional Metatron's Cube

The mismatch observed between the dodecahedron-like shape from the two-dimensional Metatron's Cube and the actual projection of the dodecahedron is based on the proportions of the regular pentagon. Assuming the dimension of the regular pentagon illustrated in Figure 14 is 1, its edge would measure 0.618... [7]

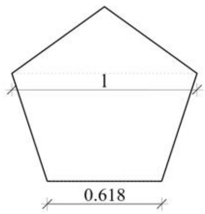

Figure 14 Proportions of the regular pentagon

When this measurement is applied to the accurate dodecahedron projection and scaled to a dimension of 1, the edge of the dodecahedron must be equal to 0.618... (Figure 15. right). Therewithal, when this measurement was applied to dodecahedron-like shape from the two-dimensional Metatron's Cube, the edge was equal to 0.5 which directly indicated that this shape does not represent a properly and accurately constructed dodecahedron (Figure 15. left). These distances were directly measured in AutoCAD software.

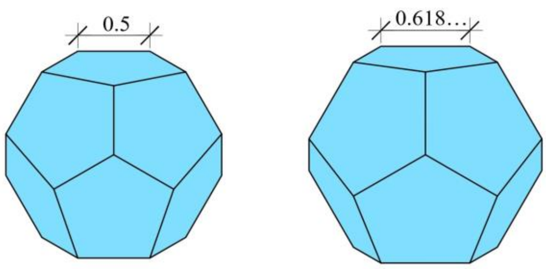

Figure 15 Differences between dodecahedron-like shape from Metatron's Cube (left) and real dodecahedron projection (right)

Based on the determination that it is impossible to construct the projection of a regular dodecahedron within the two-dimensional Metatron's Cube, it was concluded that constructing a regular dodecahedron from the three-dimensional Metatron's Cube is likewise infeasible.

4.5 Icosahedron

An icosahedron is a regular polyhedron consisting by twenty faces, each of which is an equilateral triangle. This Platonic Solid is defined by twelve vertices and thirty edges. [6]

At first, it seemed that the icosahedron's projection was observable within the two-dimensional Metatron's Cube. Upon careful analysis, it was determined that this shape is not a real projection of icosahedron. The icosahedron look-alike shape made within two-dimensional Metatron's Cube consists of two equilateral triangles with one contained within the other, along with the linkage of the centres of the outer circles with each other, and the connection of the centres of three outer circles with those of the three inner circles. This shape created in the described manner failed to accurately represent an icosahedron because the projection of an equilateral triangle from the side cannot be equal to that same equilateral triangle by its dimensions. According to this, the differences between icosahedron look-alike shape created from two-dimensional Metatron's Cube and real icosahedron are obvious (Figure 16).

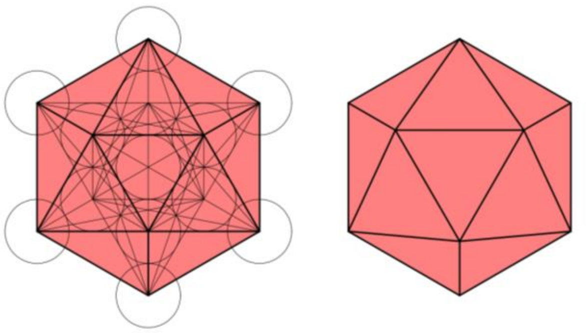

Figure 16 Differences between icosahedron look-alike shape in two-dimensional Metatron's Cube (left) and real icosahedron (right)

Due to the impossibility of constructing at least one of the projections of the icosahedron within the two-dimensional Metatron's Cube, it was concluded that it is also impossible to construct icosahedron using the three-dimensional model of the Metatron's Cube.

5. DISCUSSION

This research led to significant findings and conclusions regarding the feasibility of constructing Platonic solids utilizing the Metatron's Cube. While the hypothesis that all Platonic Solids can be constructed was supported for some solids, it was disproven for others. It was discovered that while three out of the five Platonic solids - namely the tetrahedron, hexahedron, and octahedron - can be constructed using projections found within the two-dimensional Metatron's Cube, the dodecahedron and icosahedron do not have such projections, resulting that their construction within the Metatron's Cube is impossible.

These three Platonic solids, which have been established as constructible utilizing the Metatron's Cube, have a significant impact on architectural design. Figure 17. represents great use of these solids in different architectural projects.

Figure 17 Examples of applied Platonic solids in architecture: tetrahedron (left), hexahedron (middle) and octahedron (right) www.evolo.us www.designboom.com https://vereinkunzt.at/

The application of tetrahedron, hexahedron and octahedron, but also the Metatron's Cube in architecture is extensive. Their shapes can often be used for creation of exhibition pavilions, large modular structures, spatial grid structures, or even non-functional yet aesthetically valuable sculptures.

6. CONCLUSIONS

Through exploring the spatial potential of the Metatron's Cube, it was concluded that it's not easier and more accessible method for constructing Platonic Solids. This determination primarily stems from the inability to create more complex Platonic solids such as the dodecahedron and icosahedron, coupled with the existence of easier methods for constructing simpler Platonic solids like the tetrahedron, hexahedron, and octahedron.

The paper provides space for further exploration into the application of the Metatron's Cube, particularly its three-dimensional model, offering possibilities beyond constructing three of the Platonic Solids to potentially serving as a coordinate system for diverse structures in architecture.

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Authors:

Isidora MITROVIĆ, University of Niš, Faculty of Civil Engineering and Architecture, E-mail: isidoramitrovic00@gmail.com

Prof. Sonja KRASIĆ, University of Niš, Faculty of Civil Engineering and Architecture, E-mail: krasic.sonja@gmail.com

Jovana TOMIĆ, University of Niš, Faculty of Civil Engineering and Architecture, E-mail: jovanastomic@hotmail.com

Nastasija KOCIĆ, University of Niš, Faculty of Civil Engineering and Architecture, E-mail: nastasija.kocic@gmail.com