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STUDYING THE GENERATION OF PONCELET'S CAPRICORNOID CURVE BY AN ARTOBOLEVSKII MECHANISM

Abstract: *The kinematic possibilities of a mechanism created by I. I. Artobolevskii for tracing Poncelet's capricornoid are studied. The structure of the mechanism is also studied by the contour method and the desired curve is obtained. Next, some parameters of the mechanism are changed, resulting other curves, or variants of the capricornoid with other dimensions and other positions of the two branches of the curve.*

Key words: *Poncelet's capricornoid curve, capricornoid generating mechanism*

1. INTRODUCTION

In [1] there is presented a mechanism that draws a special curve called Poncelet's capricornoid, published by himself in Paris in 1864. The name comes from the zodiacal sign of Capricorn. The same figure also appears in [3].

The capricornoid of Poncelet belongs with curves of the fourth degree. Poncelet's formula are given below:

$$\rho_D = \frac{a.b.\sin\varphi}{a+b.\cos\varphi} \qquad (1)$$

and the Cartesian equation of the curve is:

$$b^2x^2(x^2+y^2) = a^2(by - x^2 - y^2)^2 \qquad (2)$$

The curve is also presented in [2], where its parametric equations are given. This mechanism is studied below and other possibilities of it are established.

2. KINEMATIC AND STRUCTURAL SCHEME

In figure 1 we present the kinematic scheme of the mechanism, adapted according to [1].

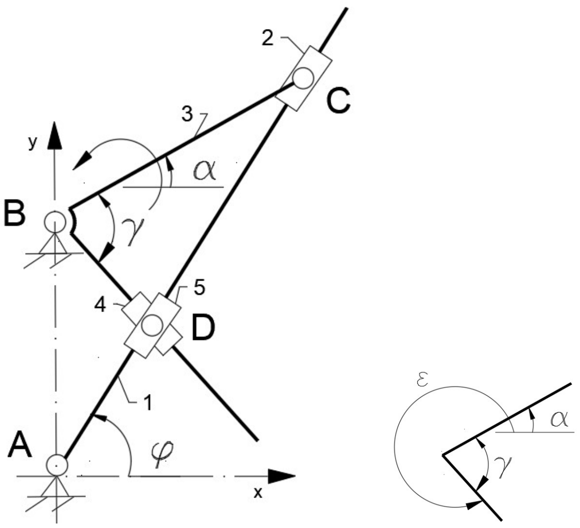

Figure 1 Kinematic scheme

The structural scheme and the decomposition into kinematic groups are given in figure 2.

The mechanism consists of the leading element CBD with rotating motion, AC dyad being RPR type and the dyad composed by elements 4 and 5 being PRP type, so the structural formula of the mechanism is R-RPR-PRP.

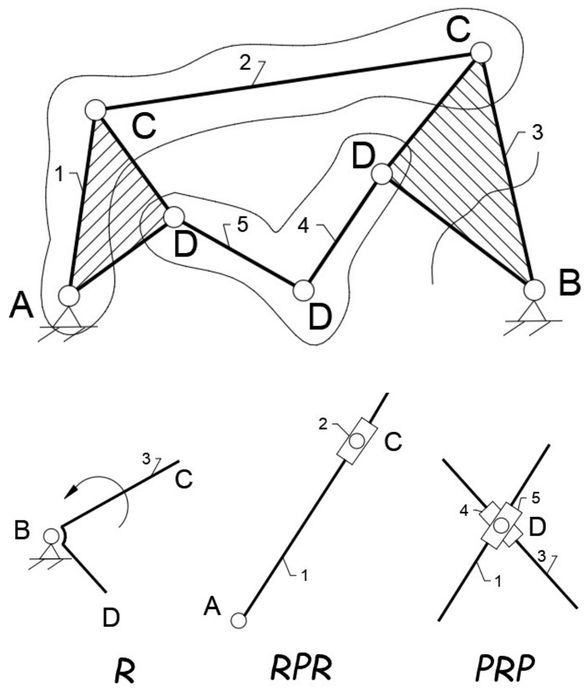

Figure 2 The mechanism structure

3. CALCULATION OF THE POSITIONS OF THE ELEMENTS

The following relations are written using the outline method:

$$X_C = AC\cos\varphi = BC\cos\alpha \qquad (3)$$
$$Y_C = AC\sin\varphi = Y_B + BC\sin\alpha \qquad (4)$$

From (3) and (4) with relations (8) and (9), φ and AC are calculated. With relations (5), (6) and (7) the BD and AD are determined according to the relations (10), (11) and (12), and with (13) and (14) the coordinates of the D tracer point are calculated.

$$X_D = X_B + BD\cos\varepsilon = AD\cos\varphi \quad (5)$$
$$Y_D = Y_B + BD\sin\varepsilon = AD\sin\varphi \quad (6)$$
$$\varepsilon = 360 - \gamma + \alpha \quad (7)$$
$$tg\ \varphi = \frac{Y_B + BC\sin\alpha}{BC\cos\alpha} \quad (8)$$
$$AC = \frac{BC\cos\alpha}{\cos\varphi} \quad (9)$$
$$tg\ \varphi = \frac{Y_B + BD\sin\varepsilon}{X_B + BD\cos\varepsilon} \quad (10)$$
$$BD = \frac{Y_B - X_B\, tg\,\varphi}{\cos(\alpha+270)\, tg\varphi - \sin\varepsilon} \quad (11)$$
$$AD = \frac{X_B + BD\cos\varepsilon}{\cos\varphi} \quad (12)$$
$$X_D = AD\cos\varphi \quad (13)$$
$$Y_D = AD\sin\varphi \quad (14)$$

4. OBTAINED RESULTS

Initial data were taken (by measuring the dimensions on the initial sketch): BC = 18 mm, AB = Y_B = 38 mm, $\gamma=90^0$.

In figure 3 we show Poncelet's capricornoid obtained for S1 = + 1, S2 = + 1, because when passing from the tangent (see Eq. (8)) to the sine and cosine, the signs (+) and (-) appear in front of the radicals.

The same curve results for S1 = -1, S2 = -1, but for S1 = -1 and S2 = + 1, the inverted curve is obtained like in figure 4.

For S1 = 1 and S2 = -1, the curve is like in figure 4.

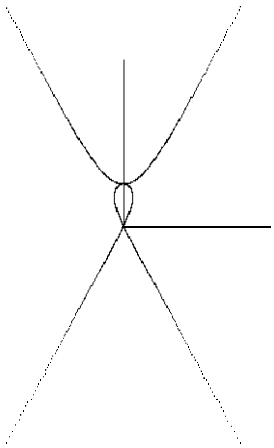

Figure 3 The capricornoid curve for S1 = + 1, S2 = + 1

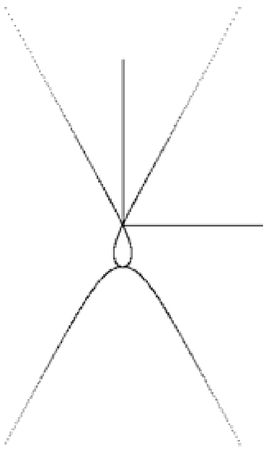

Figure 4 The capricornoid curve for S1 = - 1, S2 = + 1

Further work was done on S1 = + 1, S2 = + 1.

5. CURVES RESULTS BY MODIFICATION OF γ ANGLE

After that the angle γ was changed (figures from 5 to 14). It is found that at γ low values, the curve has two branches, completely different from capricornoid. When γ approaching 90 degrees, the curve resembles the capriconoid. Between γ= 90 and γ= 180, the curve is also a kind of capricornoid, but the upper branch changes its position. At γ= 180, as at γ= 0, the resulting curve is a circle. For γ> 180 the curve has the displaced branches, and at γ= 270 the curve is similar to the one obtained from γ= 90.

Figure 5 γ=10

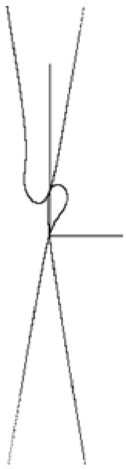

Figure 6 γ=20

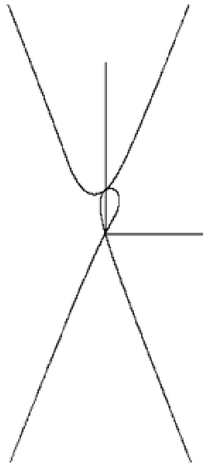

Figure 7 γ=50

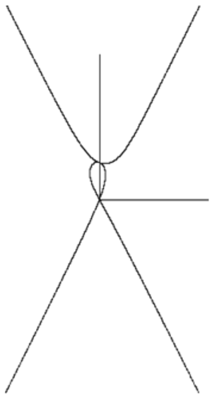

Figure 8 γ=110

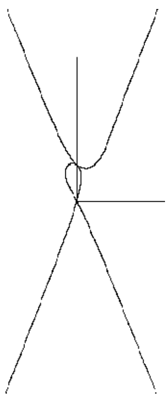

Figure 9 γ=130

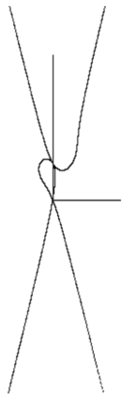

Figure 10 γ=150

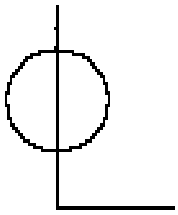

Figure 11 γ=180

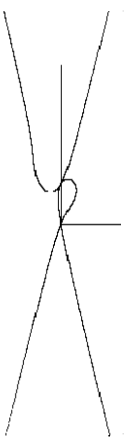

Figure 12 γ=210

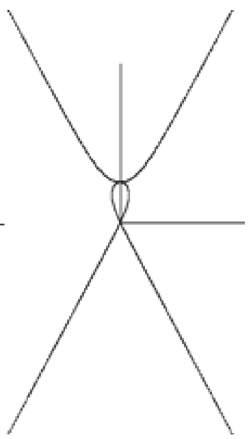

Figure 13 γ=270

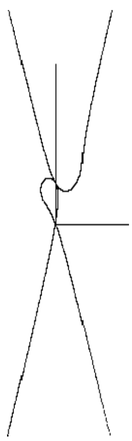

Figure 14 γ=330

6. CURVES RESULTS BY MODIFICATION OF BC AND Y_B

The initial values for BC and Y_B were modified maintaining γ= 90, resulting the curves shows in figure 15 to figure 23. It is found that at equal values for BC and Y_B the curve has a single branch. By changing BC and Y_B, intersected ovals are obtained. When BC and Y_B have close values, the branches of the curves are much flattened (figure 18). When Y_B increases the curve has the shape of the normal one, but the widths of the curves are smaller (figures 19 and 20). When BC <0, or Y_B <0 the branches of the curves change their positions between them.

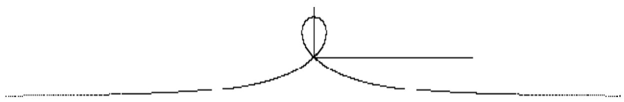

Figure 15 BC=38; Y_B=38

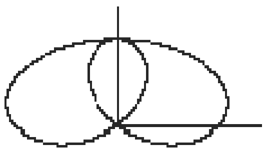

Figure 16 BC=50; Y_B=38

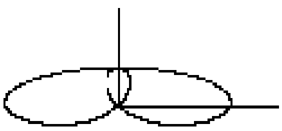

Figure 17 BC=18; Y_B=17

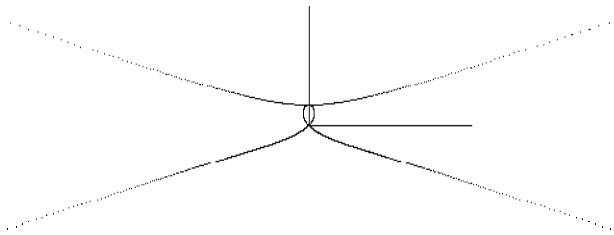

Figure 18 BC=18; Y_B=19

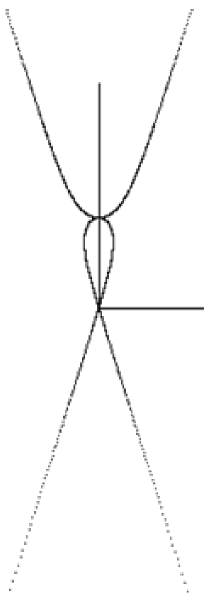

Figure 19 BC=18; Y_B=60

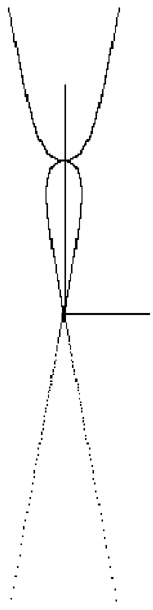

Figure 20 BC=18; Y_B=100

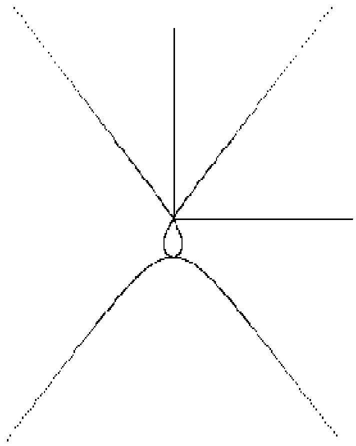

Figure 21 BC=-30; Y_B=18

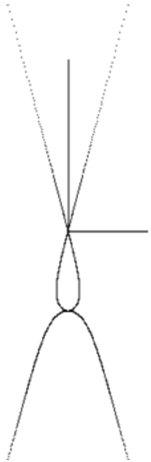

Figure 22 BC=18; Y_B=-70

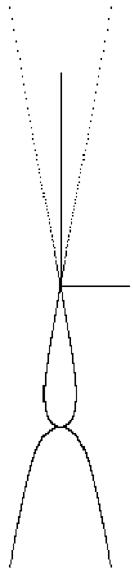

Figure 23 BC=18; Y_B=-100

Sets of values for BC and Y_B were also tried, resulting in similar curves but in other positions (figures 24 to 32). At Y_B = 0 the mechanism does not work.

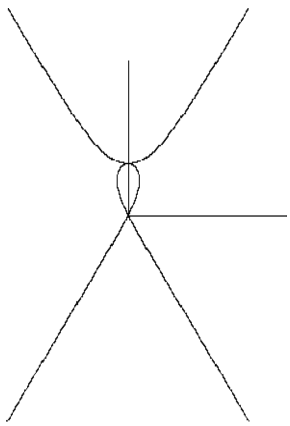

Figure 24 BC=25; Y_B=50

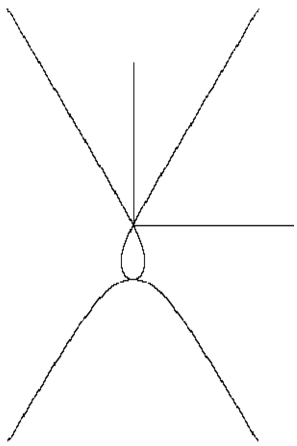

Figure 25 BC=25; Y_B=-50

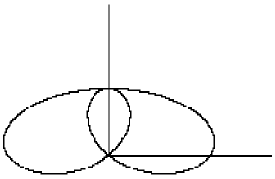
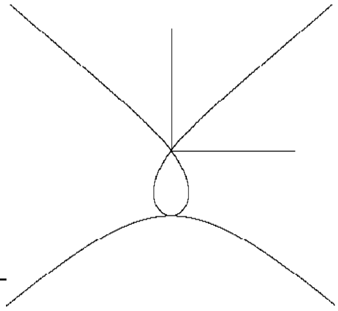

Figure 26 BC=60; Y_B=50 **Figure 27** BC=60; Y_B=-80

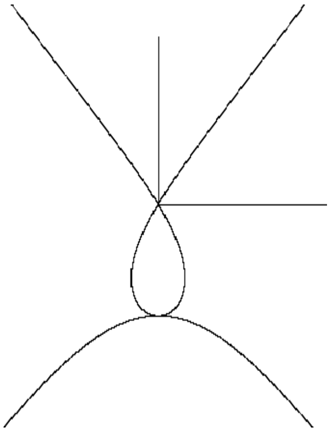
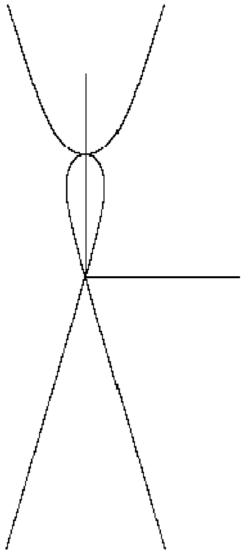

Figure 28 BC=60; Y_B=-100 **Figure 29** BC=25; Y_B=90

Further work was done on S1 = - 1, S2 = + 1 and we obtained results like in figure 30 to 32.

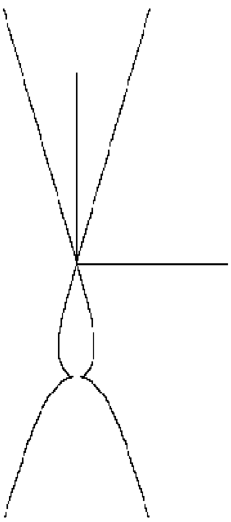
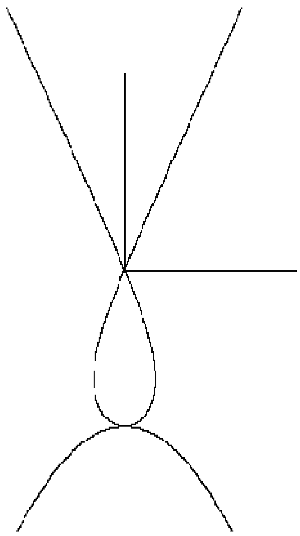

Figure 30 BC=25; Y_B=90 **Figure 31** BC=50; Y_B=120

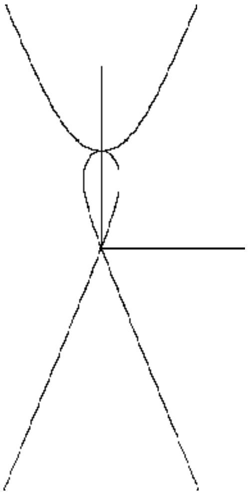

Figure 32 BC=30; Y_B=-80

7. CONCLUSION

We started from the Poncelet curve called capricornoid and from the mechanism designed by professor Artobolevskii, which was studied with relations obtained by the method of contours. The desired curve was obtained. Next, the angle of the bent bar was modified, obtaining other curves, also with two branches each, which approach the normal curve when the bending angle approaches 90 and 270 degrees. Other curves generated by this mechanism were also studied by changing the length of the crank and the order of its joint (these are the data that define the mechanism). Other shapes of the initial curve were obtained, positioned differently and deformed. At negative values of these dimensions the branches of the curves change their positions between them.

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