

## TEACHING DESCRIPTIVE GEOMETRY – CONSTRUCTING AND DEVELOPING THE SURFACES OF SOLIDS

**Abstract:** *The paper presents applications developed for teaching descriptive geometry – representation of solids and development of the surface of the solids. The purpose of the applications presented is to enable the development and improvement of the spatial ability of the students such as to understand the methods used in descriptive geometry for constructing and developing the surfaces of the solids.*

**Key words:** *Solids, projections, development of the surface, pyramid, method of revolution.*

### 1. INTRODUCTION

The paper presents applications which may be used in teaching descriptive geometry, for the understanding of the principles used for constructing and developing the surfaces of solids. The aim of these applications is to develop and improve the spatial ability of the students and, at the same time to develop the skills needed for understanding the shapes of the solids and their projections on the planes of projection.

The descriptive geometry course is usually taught in the first semester of the first year of the university studies. At the beginning of this course, the spatial abilities of the students are usually not satisfactory; they are reduced to those gained by the students in the former schools which they have attended. Plane and spatial geometry is taught only in the primary school; in the high school pupils come barely across to some elements of the plane or spatial geometry. In these conditions, descriptive geometry subject needs to be taught such as to fill the gaps in the knowledge of the students, towards the elements of the plane or spatial geometry and, at the same time, to enable the development of the spatial ability needed for understanding the methods used for constructing the projections of the elements of the plane and spatial geometry and to enable the connection with their spatial shape.

In teaching descriptive geometry, teachers need to use appropriate methods that would enable students understand the spatial shape of different types of geometrical elements as well as the representation of their orthogonal projections. Books, didactical materials, posters, as well as more attractive methods for the students - computer presentations, animations, are used on this purpose.

The applications presented in the paper are focused on the construction of the projections needed for defining a solid, as well as of the development of the surface of the considered solid. Applications are focused on simple solids placed in particular positions and they try to establish the connection between the 3D shape of the solid and its orthogonal projections and, at the same time, they try to help them find the true size of the lengths of the edges needed for developing the surface of the considered solid.

The paper continues and completes other applications which have been developed for teaching both descriptive

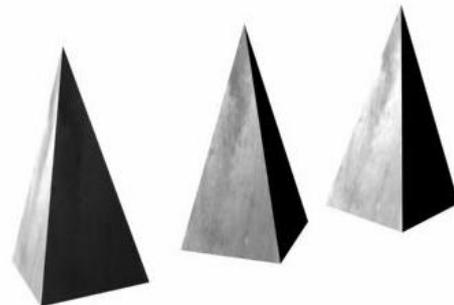
geometry and technical drawing [1], [2], [3], applications which can be easily applied in teaching any other subject involving graphical representations.

The applications designed are used for presentation (video, posters) at the classes.

### 2. CONSTRUCTING AND DEVELOPING THE SURFACE OF A RIGHT REGULAR PYRAMID

The applications presented consider right regular pyramids with the base in the horizontal plane of projection [H]. The purpose of the application is represented by the construction of the orthogonal projections of the pyramid and then, the development of the surface of the pyramid. The solid to start with is represented by a right regular quadrilateral pyramid; then, students are encouraged to consider right regular triangular pyramids and right regular hexagonal pyramids, placed in different positions relative to plane [H].

Students are shown a few possibilities to place the pyramid with the base in plane [H] (Figure 1). They need to understand the fact that, by placing a solid in a certain position toward the planes of projection, the elements needed for constructing the developed surface of the solid can be obtained at once, from its projections, or they need to be found by using appropriate methods. Students are to analyse in each case considered the elements of the solid, to decide whether they are in true size or not and if there is to apply a method for finding the true size, they need to decide which is the suitable method.



**Fig. 1** Right regular quadrilateral pyramids with the base in different positions in the horizontal plane of projection [H].

In the first case considered – case 1, the right regular quadrilateral pyramid with the base in the horizontal plane of projection [H] is placed such as the edges of the square are parallel to axes (Ox) and (Oy), respectively. Figure 2 indicates the way in which the projections are constructed, while Figure 3 shows the projections of the pyramid on the three planes of projection [H], [V] and [L].

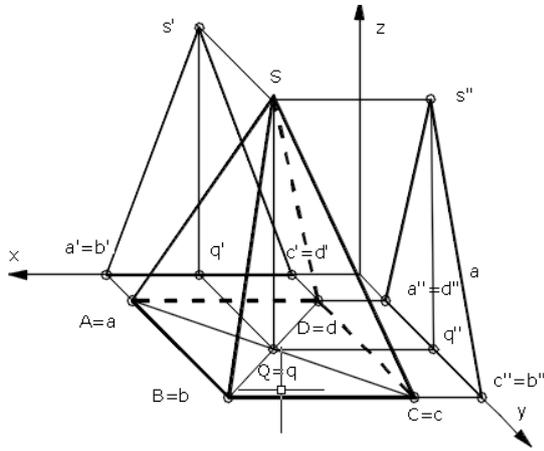


Fig. 2 Right regular quadrilateral pyramid with the base in the horizontal plane of projection [H]-case 1.

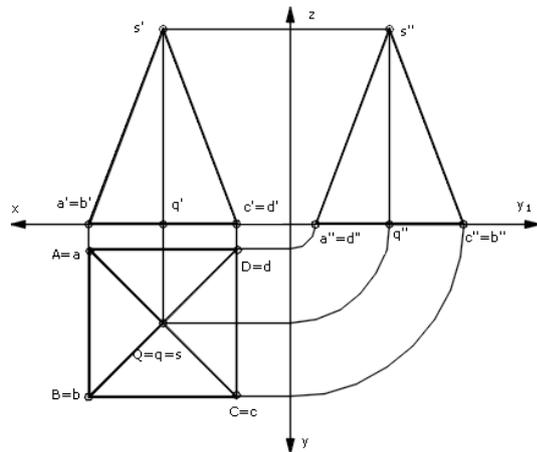


Fig. 3 Projections of a right regular quadrilateral pyramid with the base in the horizontal plane of projection [H]-case 1.

Students are invited to analyse the projections and to decide whether one can find in true size the elements needed to construct the developed surface of the pyramid. They should realize that the square representing the base of the pyramid is represented in true size in plane [H], but the side edges of the pyramid, which are all of the same size, are not in particular positions (parallel to a plane of projection) and so, one needs to find their true sizes.

A first method used for finding the true size of the side edges of the pyramid is the method of revolution [4], [5], [6]. According to this method, a side edge of the pyramid which is in a general position -  $|SA|$  is revolved about an axis of revolution represented by the height of the square  $|SQ|$  until it is brought into a

position parallel to plane [V] and becomes a vertical line  $|SE|$ , its true size being represented by its vertical projection  $|s'e'|$ :  $\|SA\| = \|SE\| = \|s'e'\|$  (Figure 4 and Figure 5). Since now all needed sizes are known, one can construct the developed surface of the pyramid.

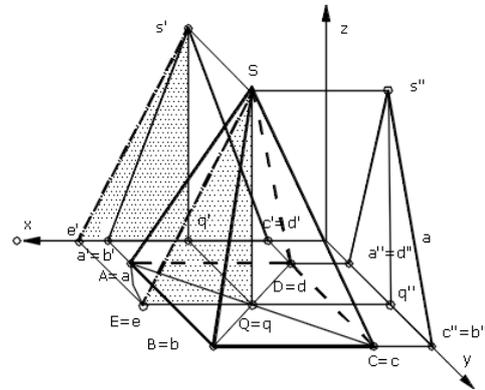


Fig. 4 Application of the method of revolution for finding the true size of the side edge  $|SA|$ .

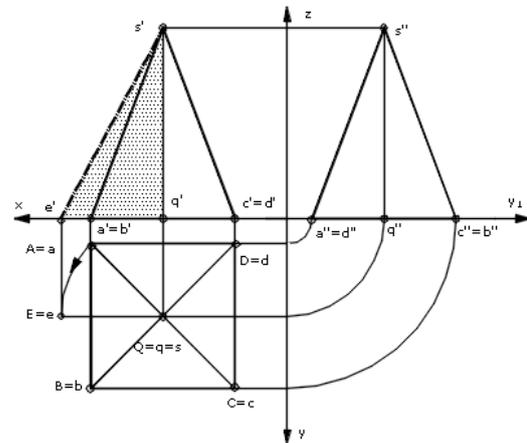


Fig. 5 Application of the method of revolution for finding the true size of the side edge  $|SA|$  - orthogonal projection.

Other method which could be used for finding the true size of the side edge of the pyramid is to revolve a side face of the pyramid  $[SAB]$ , about its base located in plane [H], until this face is contained by plane [H],  $[S_0AB] = [H]$  (Figure 6 and Figure 7).

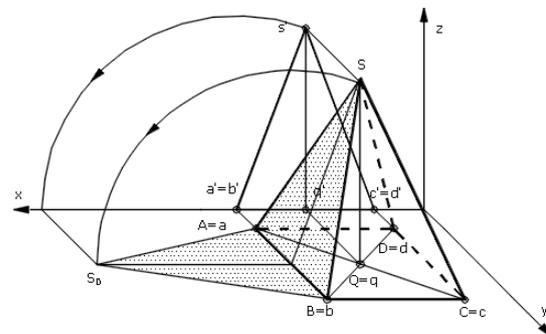
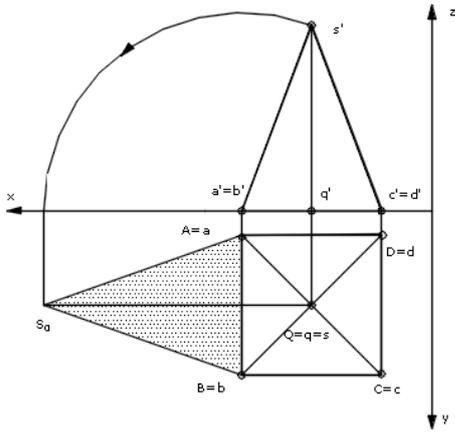
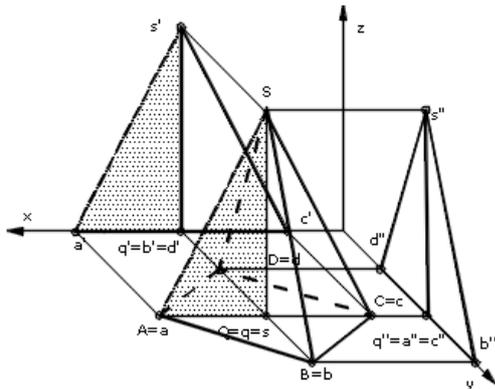


Fig. 6 Revolving the side face of a right regular quadrilateral pyramid and placing it in plane [H].

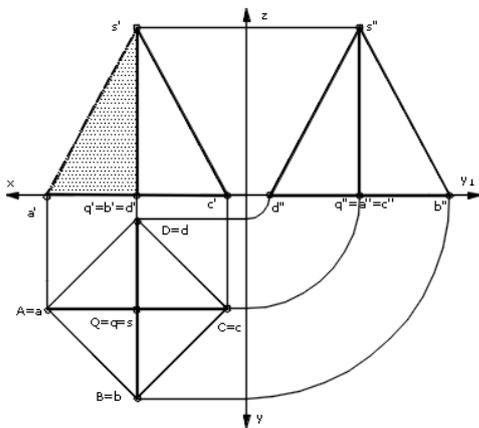


**Fig. 7** Orthogonal projection showing the revolution of the side face of a right regular quadrilateral pyramid on plane [H].

In the second case considered – case 2, the right regular quadrilateral pyramid with the base in the horizontal plane of projection [H] is placed such as the diagonals of the square are parallel to axes (Ox) and (Oy), respectively. Figure 8 indicates the 3D drawing of the solid and its orthogonal projections, while Figure 9 shows the projections of the pyramid on the three planes of projection [H], [V] and [L].



**Fig. 8** Right regular quadrilateral pyramid with the base in the horizontal plane of projection [H]-case 2.

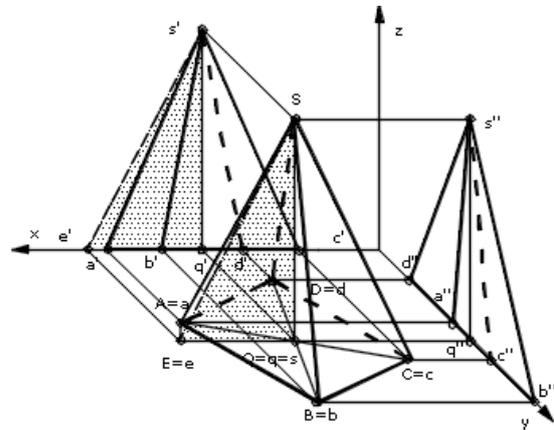


**Fig. 9** Projections of a right regular quadrilateral pyramid with the base in the horizontal plane of projection [H]-case 2

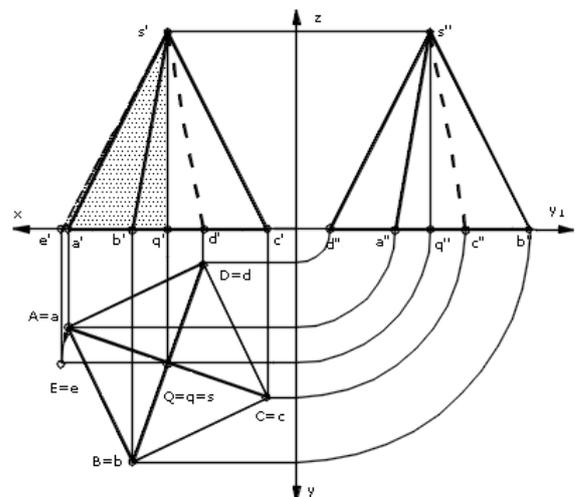
In this case, the side edges of the pyramid  $|SA|$  and  $|SC|$  are already in particular positions, parallel to the vertical plane of projection [V], and so the true size of the length of the side edge of the pyramid is known,  $\|SA\| = \|s'a'\|$ .

In the third case considered – case 3, the right regular quadrilateral pyramid with the base in the horizontal plane of projection [H] is placed in an arbitrary position, such as neither the edges nor the diagonals of the square are parallel to axes (Ox) or (Oy). Figure 10 indicates the 3D construction of the solid and its orthogonal projections and Figure 11 shows the orthogonal projections of the pyramid on the three planes of projection [H], [V] and [L].

In order to find the true size of the side edges of the pyramid, the method of revolution is used. A side edge of the pyramid which is in a general position -  $|SA|$  is revolved about the axis of revolution  $|SQ|$ , until it is brought into a position parallel to plane [V] and becomes a vertical line  $|SE|$ , its true size being represented by its vertical projection  $|s'e'|$ :  $\|SA\| = \|SE\| = \|s'e'\|$ .



**Fig. 10** Right regular quadrilateral pyramid with the base in the horizontal plane of projection [H]-case 3.



**Fig. 11** Projections of a right regular quadrilateral pyramid with the base in the horizontal plane of projection [H]-case 3.

The developed surface of the pyramid can be constructed, with all elements of the pyramid in true sizes, by using one of the two methods described in Figure 12, a, b and c (unfolded surface in star shape), or Figure 13, a and b (unfolded surface in fan shape) [4], [5], [6].

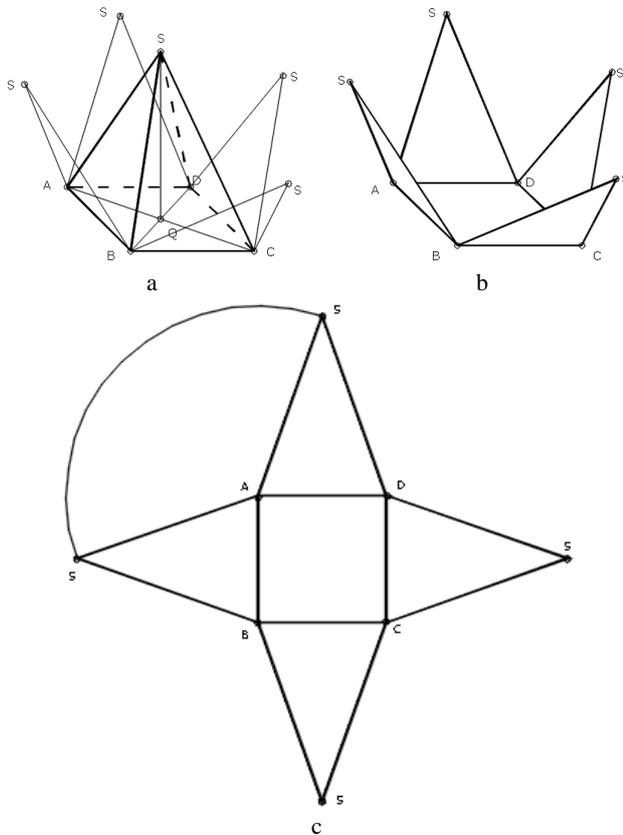


Fig. 12 Method for constructing the developed surface of the right regular quadrilateral pyramid - star shape.

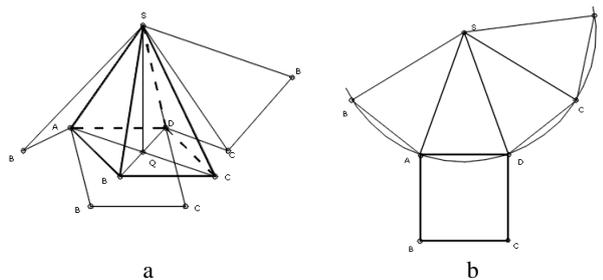


Fig. 13 Method for constructing the developed surface of the right regular quadrilateral pyramid - fan shape.

With the applications presented above, students understand more rapidly the issues needed for constructing the orthogonal projections of a given solid, they realise the reasons why proper methods are used to find the true sizes of spatial elements.

Although 3D drawings seem to be difficult to realize, by constructing these drawings in parallel with the orthogonal projections of the solid, students seem to develop the necessary skills for making the connection between the spatial shape of a part and its appropriate orthogonal projections.

### 3. CONCLUSIONS

The paper presents applications which are designed in order to develop the spatial skills of the students and to help them increase their knowledge towards the modelling of the parts. They help them understand the connection between 3D representation of a solid and its orthogonal projections and they create the premises for the development of the spatial skills of the students.

The applications developed in the paper can be applied in teaching descriptive geometry, as well as technical drawing.

### REFERENCES

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